A fundamental quantity which can be calculated is the temperature on the Hugoniot. From the first TdS equation, assume the functional form $S = S(T, V)$ for entropy so that

$$TdS = T\left(\frac{\partial S}{\partial T}\right)_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV \ . \tag{56}$$

Since

$$T(\partial S/\partial T)_V = C_V$$

and

$$(\partial S/\partial V)_T = \beta B_T = kC_V$$

then Eq. (56) can be expressed as

$$TdS = C_V dT + kTC_V dV \ . \tag{57}$$

Since $dS = 0$ on an isentrope, then Eq. (57) reduces to a differential equation for the temperature

$$dT/T = kdV \ . \tag{58}$$

The solution is

$$T = T_i e^{k\alpha} \tag{59}$$

where $\alpha = V_0 - V$ and T_i is some initial temperature on the isentrope. At the point of intersection of the isentrope and the Hugoniot curve, $\alpha = \alpha_H$ and the temperature T refers to the temperature on the Hugoniot. The initial temperature T_i is calculated from the second law of thermodynamics. Fig. 7 illustrates the method. From the second law, the change in energy along an isentrope is given by

$$dE = C_V dT$$

and after integrating, the expression becomes

$$E_i - E_0 = C_V(T_i - T_0) \ .$$

The reference energy state at the foot of the Hugoniot labeled E_0 is defined to be zero and T_0 is room temperature (approximately 300°K).